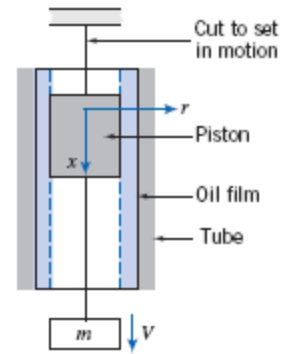


Problem 2.48

[Difficulty: 2]

2.48 A 73-mm-diameter aluminum ($SG = 2.64$) piston of 100-mm length resides in a stationary 75-mm-inner-diameter steel tube lined with SAE 10W-30 oil at 25°C. A mass $m = 2$ kg is suspended from the free end of the piston. The piston is set into motion by cutting a support cord. What is the terminal velocity of mass m ? Assume a linear velocity profile within the oil.



Given: Flow data on apparatus

Find: The terminal velocity of mass m

Solution:

Given data: $D_{\text{piston}} = 73\text{ mm}$ $D_{\text{tube}} = 75\text{ mm}$ $\text{Mass} = 2\text{ kg}$ $L = 100\text{ mm}$ $SG_{\text{Al}} = 2.64$

Reference data: $\rho_{\text{water}} = 1000 \cdot \frac{\text{kg}}{\text{m}^3}$ (maximum density of water)

From Fig. A.21, the dynamic viscosity of SAE 10W-30 oil at 25°C is: $\mu = 0.13 \cdot \frac{\text{N}\cdot\text{s}}{\text{m}^2}$

The terminal velocity of the mass m is equivalent to the terminal velocity of the piston. At that terminal speed, the acceleration of the piston is zero. Therefore, all forces acting on the piston must be balanced. This means that the force driving the motion (i.e. the weight of mass m and the piston) balances the viscous forces acting on the surface of the piston. Thus, at $r = R_{\text{piston}}$:

$$\left[\text{Mass} + SG_{\text{Al}} \cdot \rho_{\text{water}} \cdot \left(\frac{\pi \cdot D_{\text{piston}}^2 \cdot L}{4} \right) \right] \cdot g = \tau_{rz} \cdot A = \left(\mu \cdot \frac{d}{dr} V_z \right) \cdot (\pi \cdot D_{\text{piston}} \cdot L)$$

The velocity profile within the oil film is linear ...

Therefore
$$\frac{d}{dr} V_z = \frac{V}{\left(\frac{D_{\text{tube}} - D_{\text{piston}}}{2} \right)}$$

Thus, the terminal velocity of the piston, V , is:

$$V = \frac{g \cdot (SG_{\text{Al}} \cdot \rho_{\text{water}} \cdot \pi \cdot D_{\text{piston}}^2 \cdot L + 4 \cdot \text{Mass}) \cdot (D_{\text{tube}} - D_{\text{piston}})}{8 \cdot \mu \cdot \pi \cdot D_{\text{piston}} \cdot L}$$

or
$$V = 10.2 \frac{\text{m}}{\text{s}}$$

